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Incorporating non-Gaussian observation errors into variational methods

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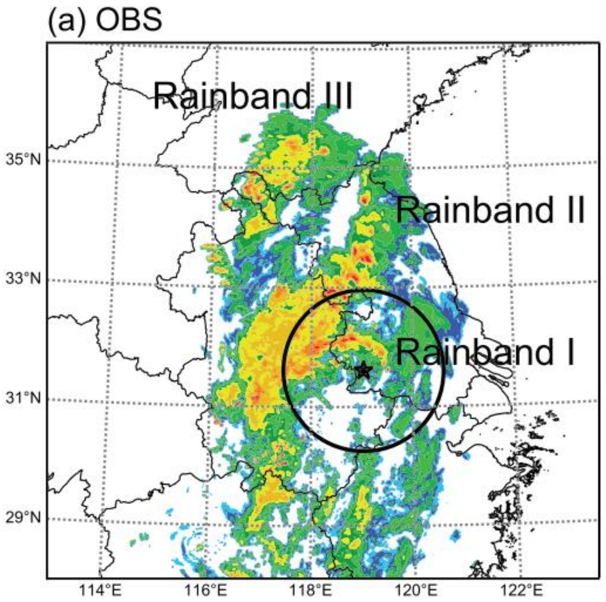
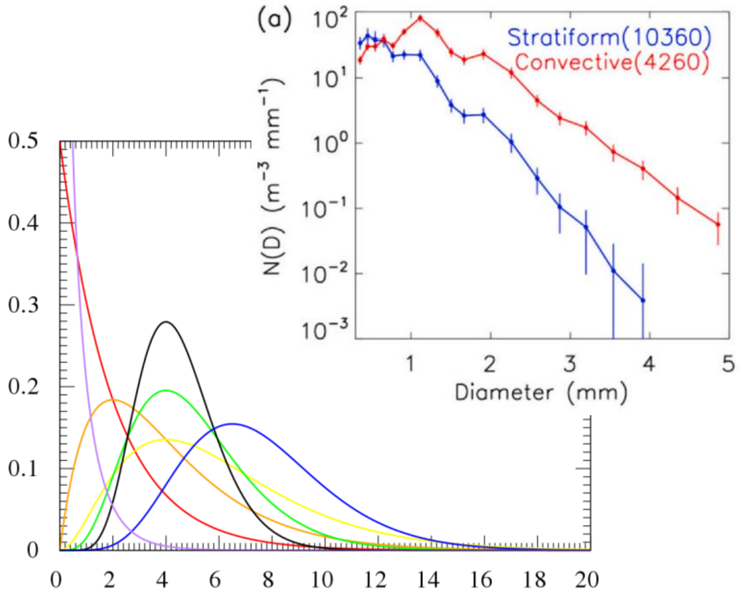
The observation errors can be quite non-Gaussian due to the errors in the observation operator

- Consider a perfect model state and a perfect observation:

$$y^{truth} \neq H\left(x^{truth}\right)$$

- The errors in the observation operator H come from
  - e.g., inappropriate assumptions in H





figures from Wang et al. (2020); Thomas et al. (2021)

<u>Example</u>: cloudy radiances

$$H = \left(RTM \circ M_{t_{initial} \rightarrow t_{obs}}\right)$$

The error can be quite non-Gaussian when predicting clouds (e.g., due to the displacement error)

How do we incorporate non-Gaussian errors into DA?

- Nonlinear DA methods, e.g., Particle Flow Filter *Daum and Huang (2011); Liu and Wang (2016); Pulido and van Leeuwen (2019); Hu and van Leeuwen (2021)*

- Variational methods
  - Derive the cost-function for the non-Gaussian pdf
  - Examples: varQC *Anderson and Järvinen (1999)*, log-normal *Fletcher and Zupanski (2006)*, Huber norm *Huber (1972); Tavolato and Isaksen (2015)*

- These methods require a prior assumption of the parametric form of the error distribution.

- **Motivation**: to propose a method that can incorporate non-Gaussian errors into the variational methods in a more general way

The "evolving-Gaussian" method for the variational methods

- Key idea: approximate the cost-function iteratively using Gaussian errors.

$J^{NG}(x)$ the cost-function from an arbitrary non-Gaussian pdf

$J^{NG}(x_0)$ —— find a Gaussian G_0 s.t. $\nabla J^{NG}(x_0) = \nabla J^{G_0}(x_0)$ ——→ $J^{G_0}(x_0)$

the first-guess (background) ↗ x_0

update the state ↓

$J^{NG}(x_1)$ —— find G_1 s.t. $\nabla J^{NG}(x_1) = \nabla J^{G_1}(x_1)$ ——→ $J^{G_1}(x_1)$

⋮

⋮

How do we find “the Gaussian(s)” in the evolving-Gaussian method?

- Assumptions:
  - Independent observation errors ⇒ the observation error pdf reduces to a 1-D pdf
  - We have access to the 1-D observation error pdf, denoted as $f_{\varepsilon^o}\big(y - H(x)\big)$

- Consider the cost-function for one observation (at the *i* th iteration):

$$J^{NG}(x_i) = J_b(x_i) + J_o^{NG}(x_i)$$

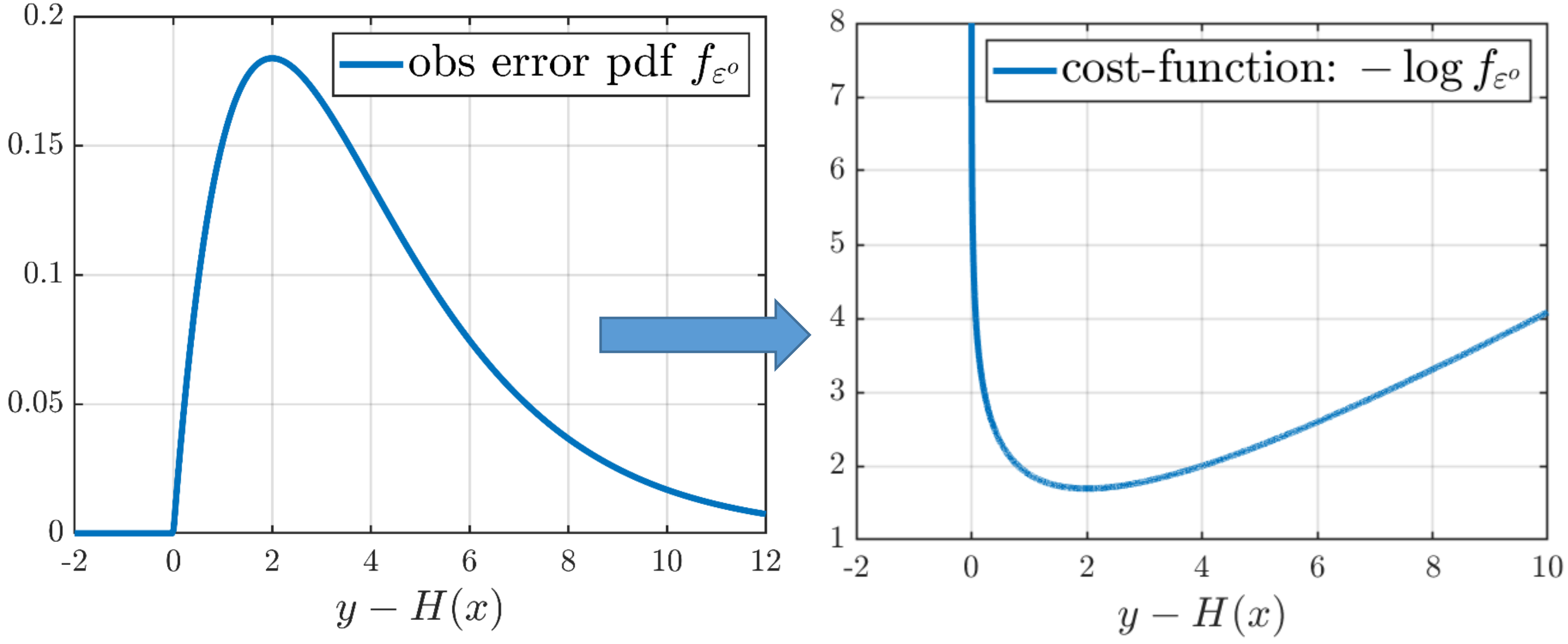
$$J_o^{NG}(x_i) = -\log f_{\varepsilon^o}\left(y - H(x_i)\right)$$

$$J_o^{G_i}(x_i) = \frac{1}{2}\left(\frac{H(x_i) - y + \mu_i}{\sigma_i}\right)^2$$

set μ_i = the mode of f_{ε^o}

and solve σ_i by $\nabla J_o^{NG}(x_i) = \nabla J_o^{G_i}(x_i)$

An example of finding “the Gaussian(s)” for a Gamma pdf



non-Gaussian obs error pdf

$$f_{\varepsilon^o}\big(y - H(x)\big)$$

non-Gaussian cost-function

$$J_o^{NG} = -\log f_{\varepsilon^o}$$

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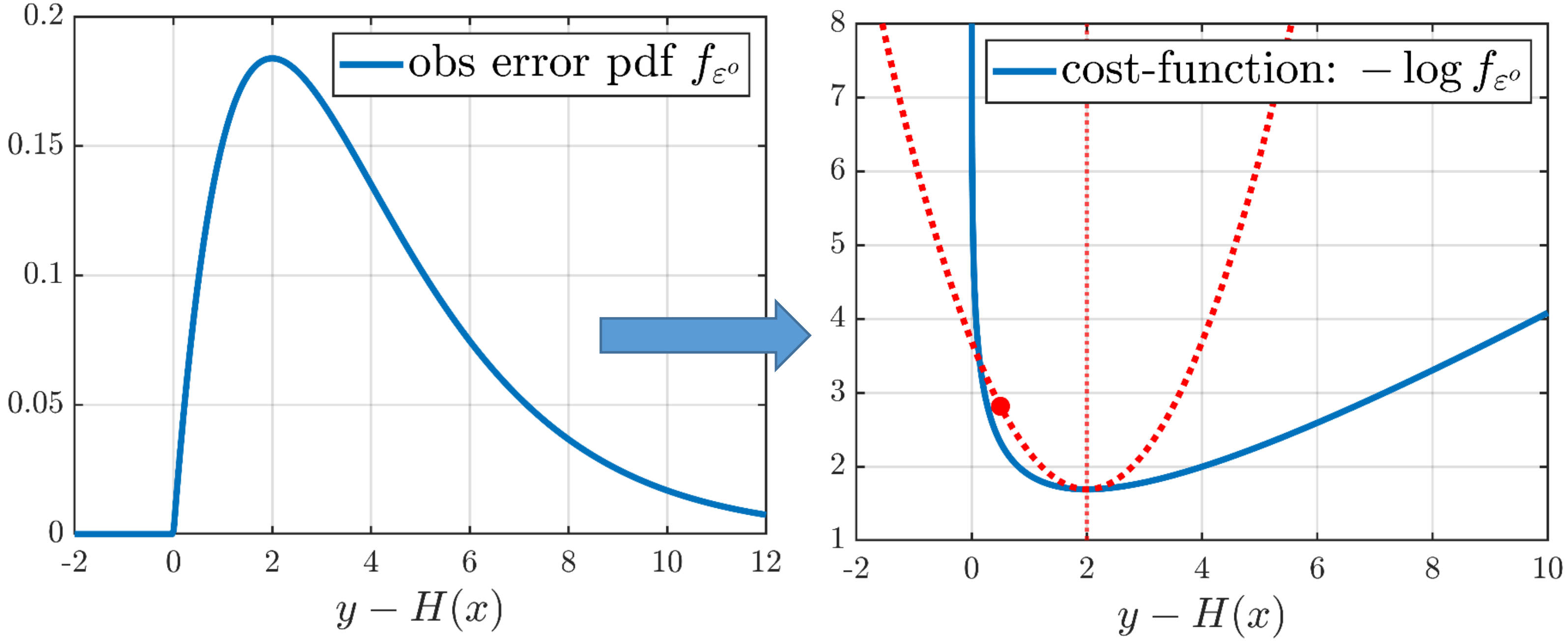
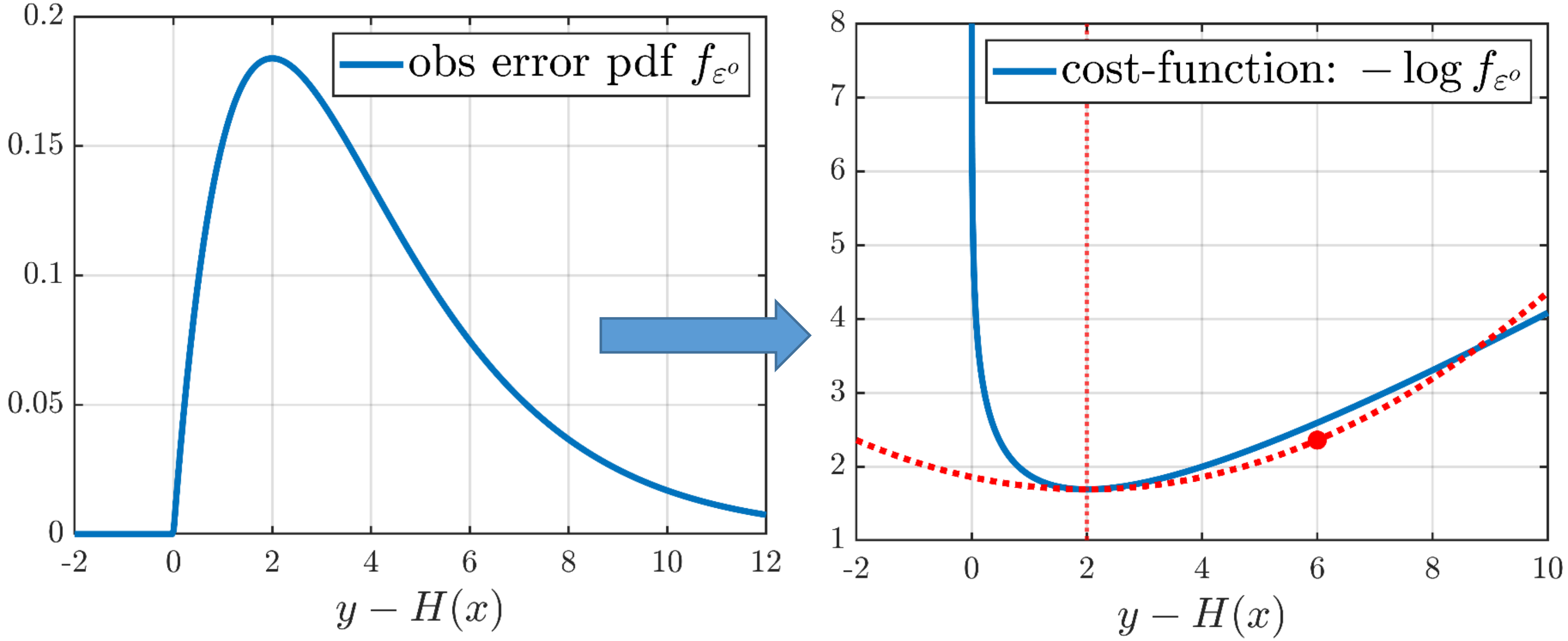


non-Gaussian obs error pdf $f_{\varepsilon^o}\big(y - H(x)\big)$

(blue) non-Gaussian cost-function $J_o^{NG} = -\log f_{\varepsilon^o}$

(red) Gaussian cost-function $J_o^{G_i}$ when $y - H(x) = 0.5$

An example of finding “the Gaussian(s)” for a Gamma pdf

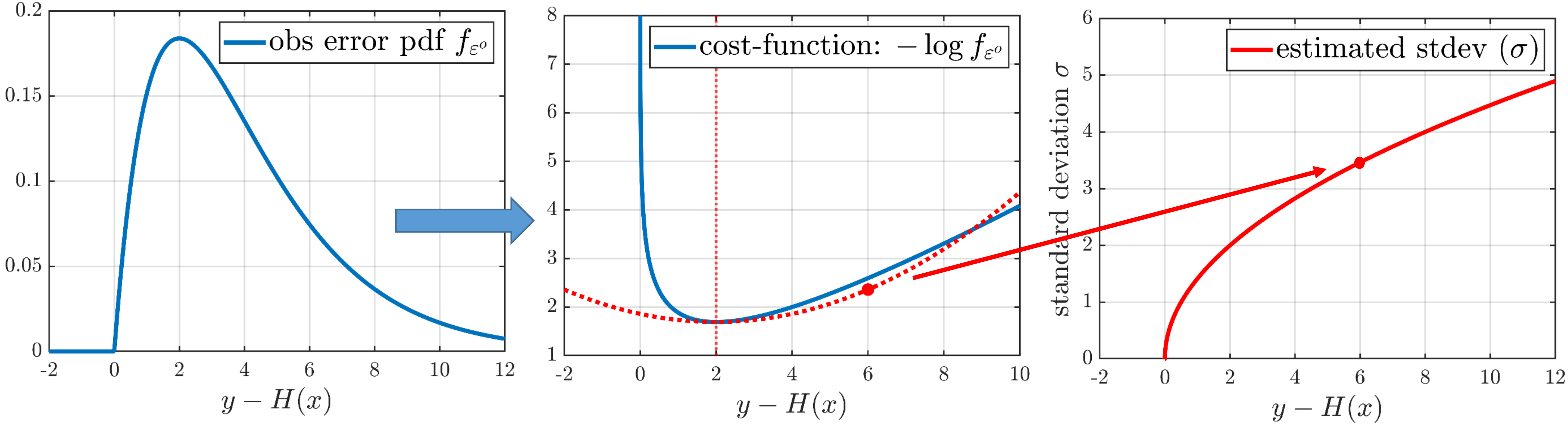


non-Gaussian obs error pdf
$f_{\varepsilon^o}\big(y - H(x)\big)$

(blue) non-Gaussian cost-function $J_o^{NG} = -\log f_{\varepsilon^o}$

(red) Gaussian cost-function $J_o^{G_i}$ when $y - H(x) = 6.0$

An example of finding “the Gaussian(s)” for a Gamma pdf



non-Gaussian obs error pdf

$f_{\varepsilon^o}\big(y - H(x)\big)$

non-Gaussian cost-function

$J_o^{NG} = -\log f_{\varepsilon^o}$

Summary of the Gaussian(s)

$\sigma\big(y - H(x)\big)$

Remarks of the evolving-Gaussian method

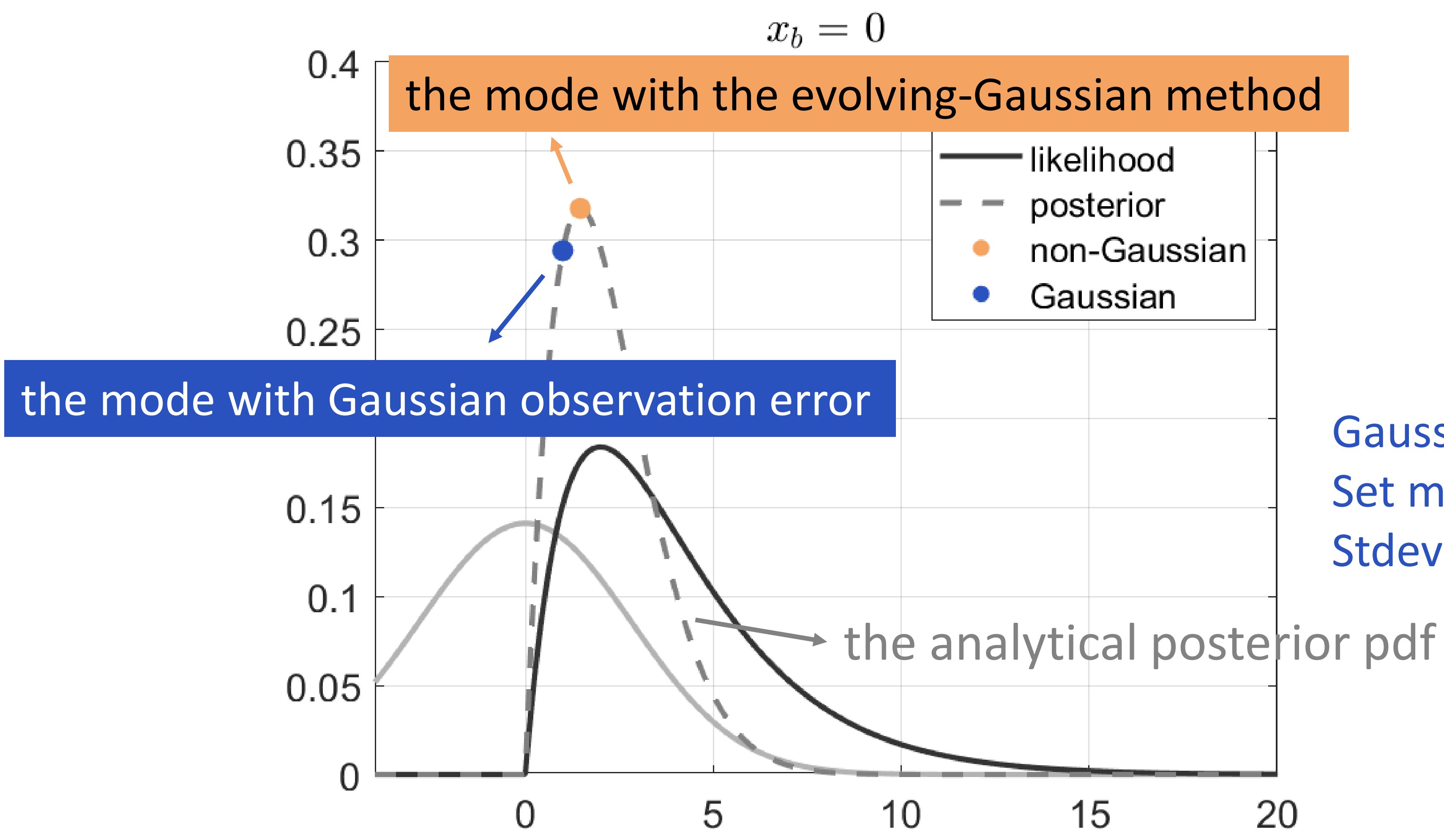
- Easy: the cost-function remains the same, no need to change the cost-function at all.

- Efficient: all the work can be done offline, only need a lookup table $\sigma\big(y - H(x)\big)$

- In theory, we need to evolve the Gaussian for every inner and outer loops.
  - need to evaluate H for each inner loop

- What if we **only** evolve the Gaussian in the outer loop?
  - A similar idea to linearize H in each outer loop in the incremental (3)4D-Var
  - The cost-function remains quadratic in the inner loops

An idealized test for the evolving-Gaussian method in the outer loop

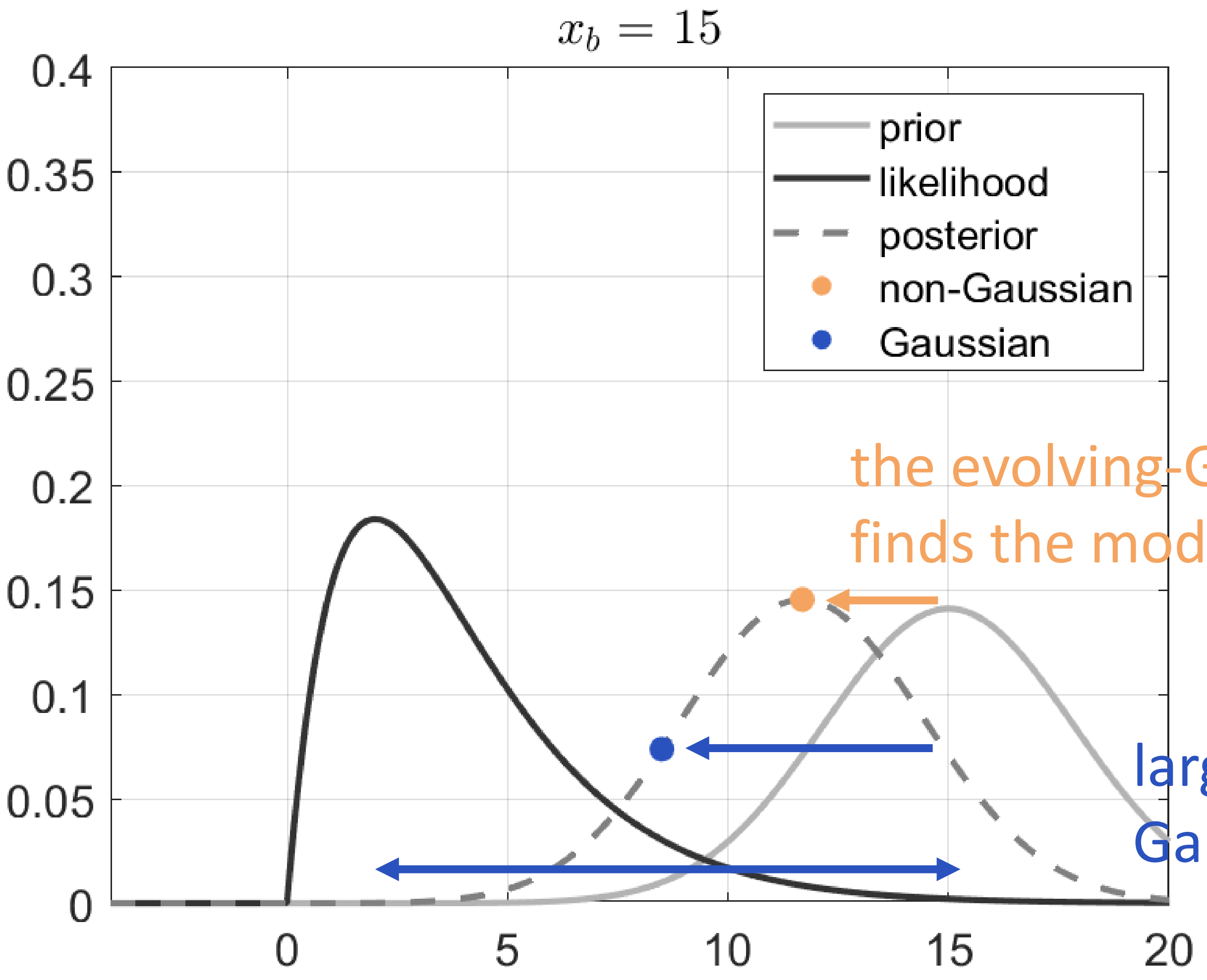


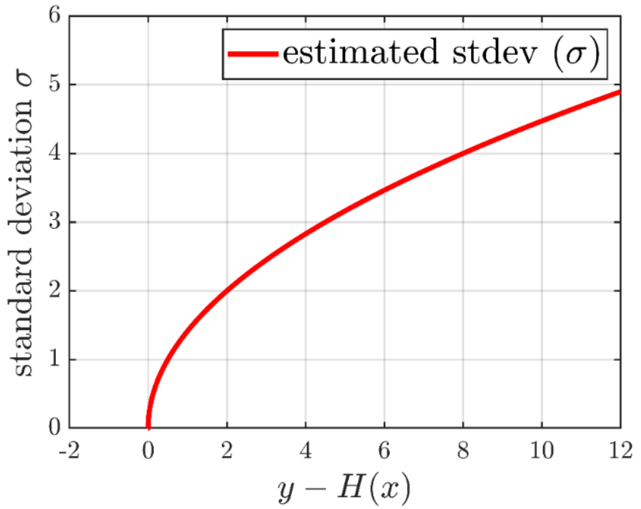
Gaussian experiment:

Set mean = mode of non-Gaussian pdf

Stdev = stdev of the non-Gaussian pdf

What can we learn from the idealized experiment?





the evolving-Gaussian method accurately finds the mode of the posterior pdf

large discrepancy between O and B

Gaussian solution: too large increment

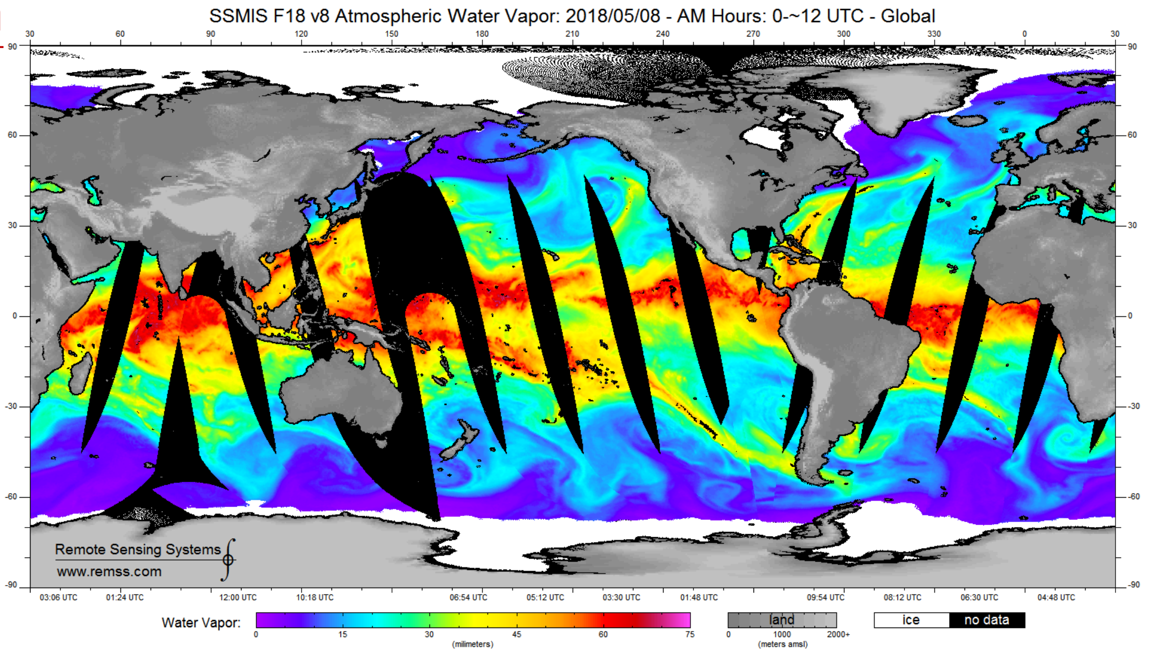
An implementation for the all-sky microwave radiance in IFS-ECMWF

- Selected SSMIS channels: ch12 (19H), ch13 (19V), ch14 (22V), ch16 (37V), ch17 (91V)
- The assumptions for the assimilation of the all-sky microwave radiances:
  - A ”symmetric cloud”-dependent **Gaussian** observation error model

non-Gaussian

$$\overline{C_{37}} = \frac{C^{o}_{37} + C^{b}_{37}}{2}$$

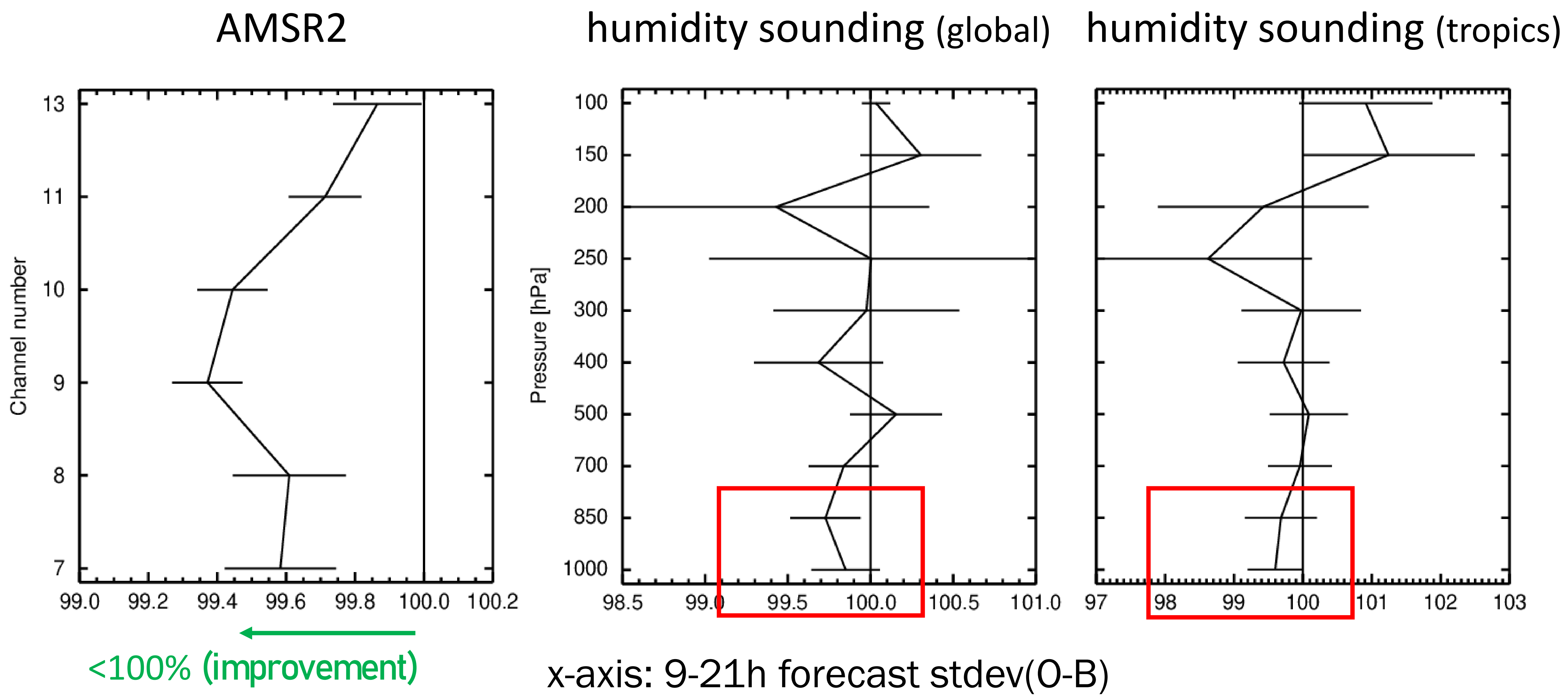
Geer and Bauer (2011)



  - The O-B (innovation) pdf is assumed to be the observation error pdf*

*(*subtracting the background error from O-B does not improve the results)*

Preliminary results suggest lower tropospheric water vapor, cloud and precipitation are improved, especially in the tropics.



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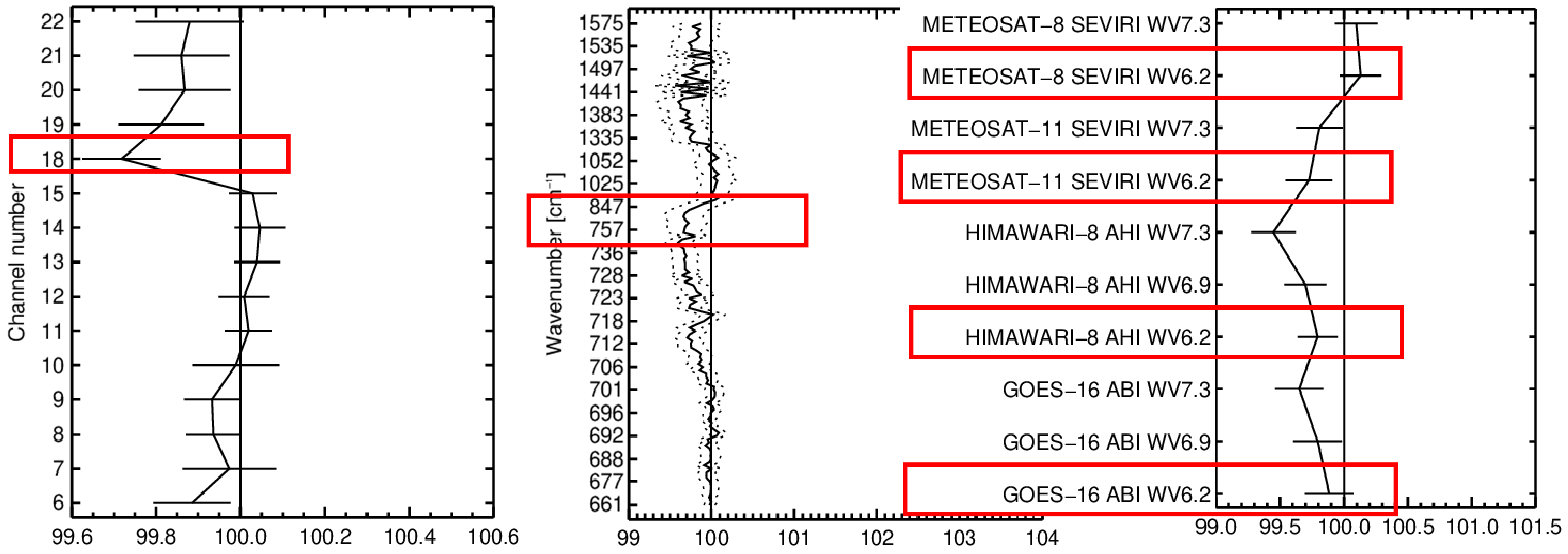
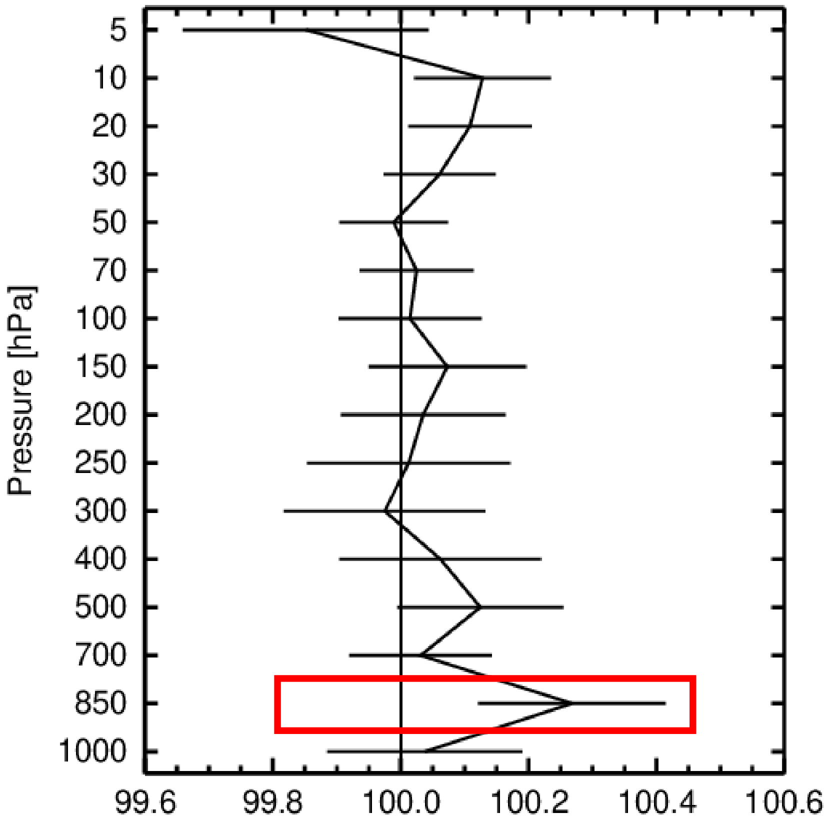
ATMS | CRIS | Geostationary satellites



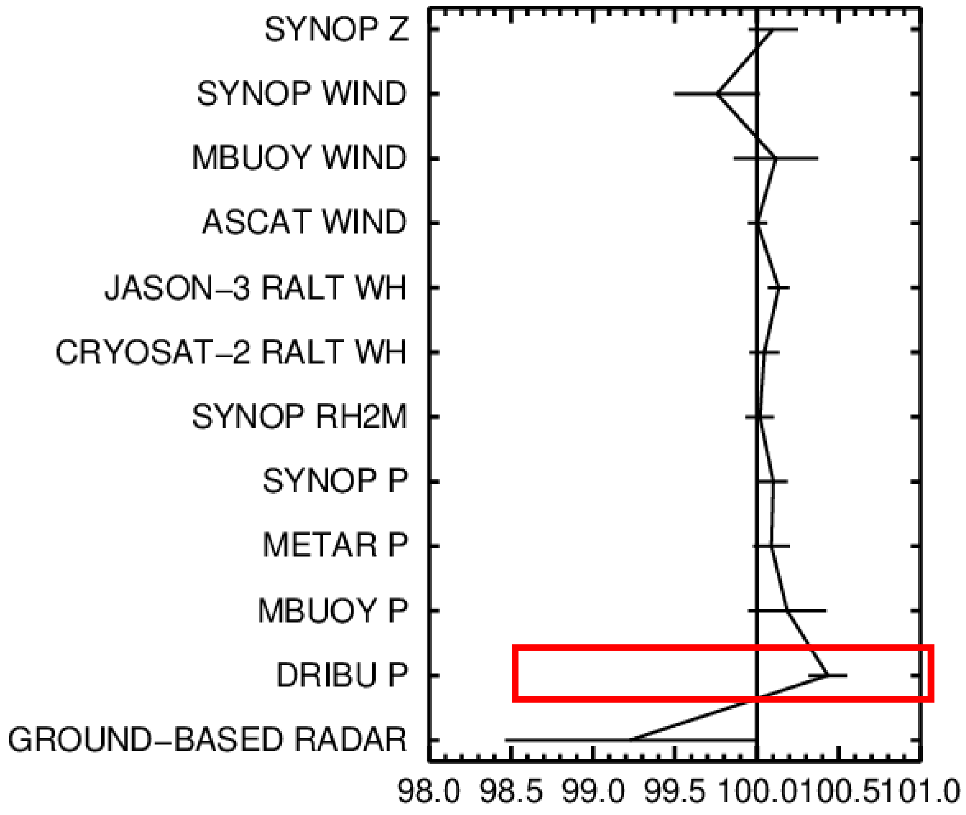
x-axis: 9-21h forecast stdev(O-B)

There are degradations in temperature and surface pressure

temperature sounding



surface observations



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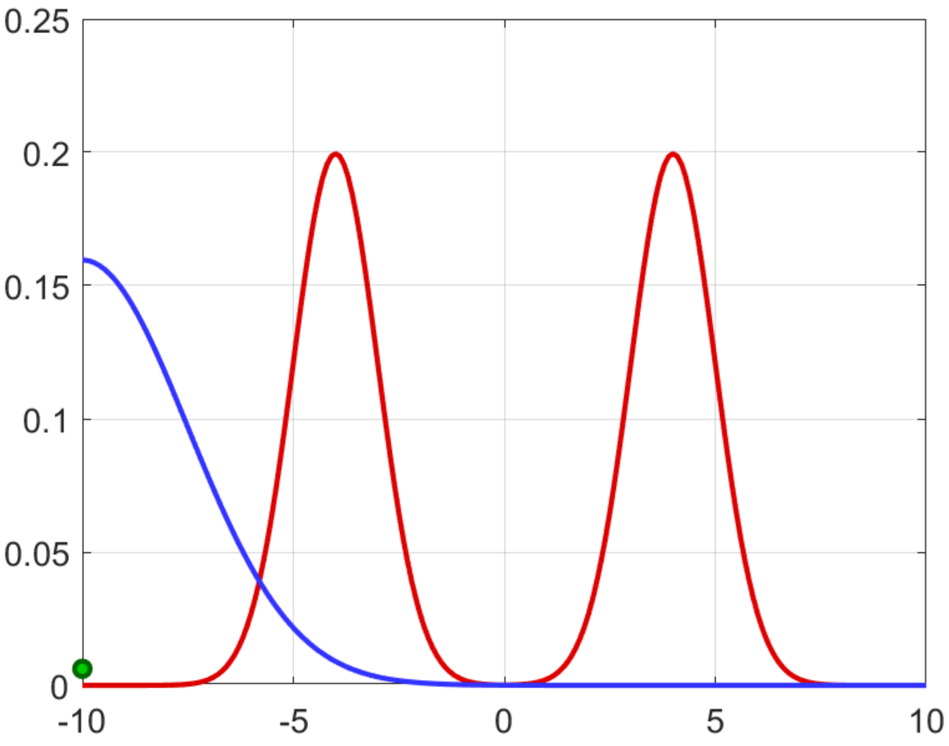
Deconvolution-based Observation Error Estimation (DOEE)

Hu, van Leeuwen and Geer (under review)

- Currently, the innovation pdf is assumed to be the observation error pdf (for the all-sky microwave radiances).

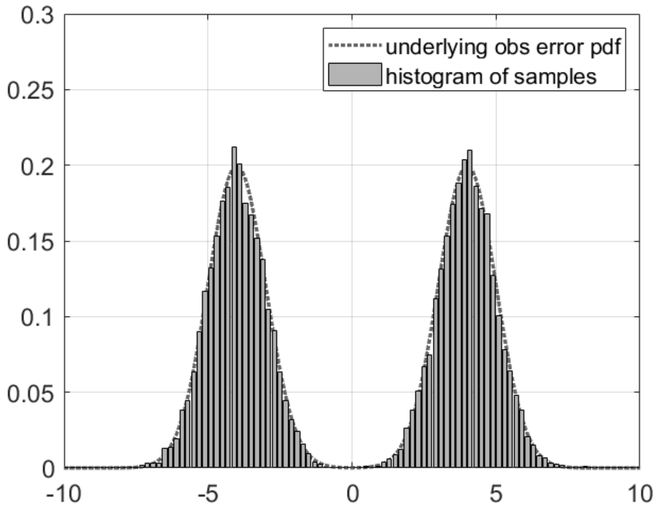
How do we identify the pdf of an arbitrary non-Gaussian observation error?

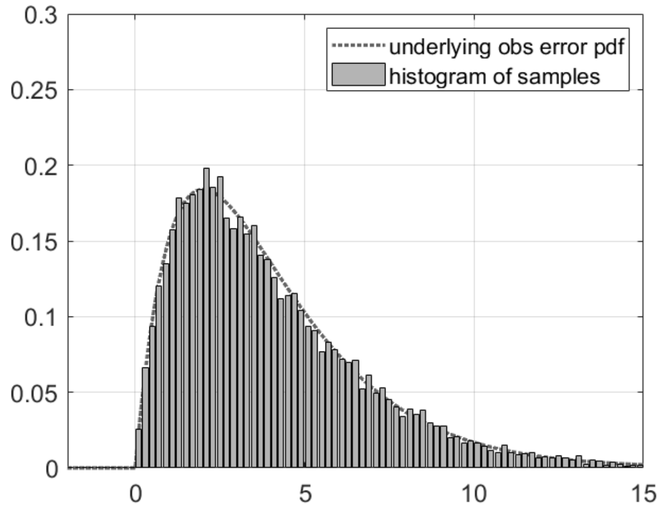
- DOEE can be used to find a *non-parametric* observation error pdf:



$$D = Y - H(X) = \varepsilon^o - \left[H(X) - H\left(X^{truth}\right)\right]$$

$$f_D = \left(f_{\varepsilon^o} * f_{H(X)-H\left(X^{truth}\right)}\right)$$





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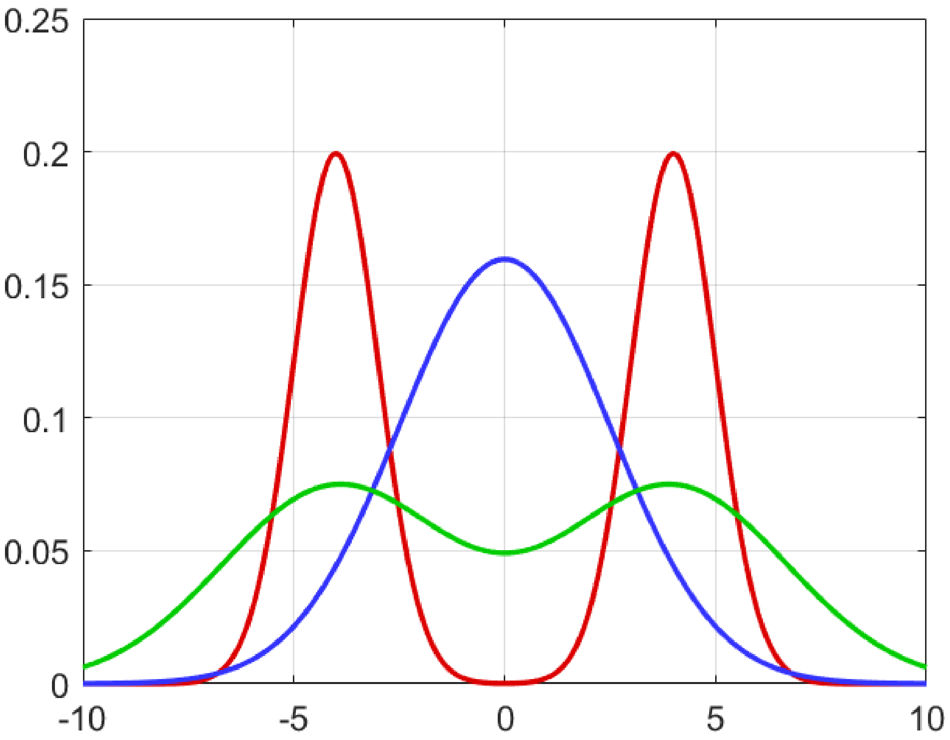

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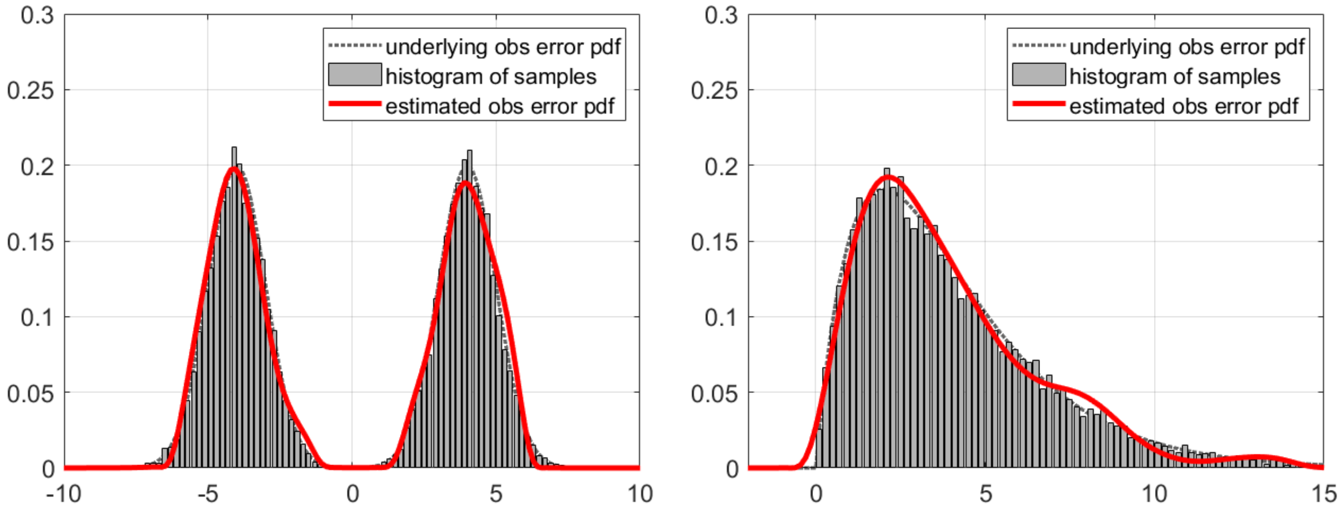

Future work and possible applications

- Incorporate non-Gaussian observation error for more observation types.
- The evolving-Gaussian method for the inner-loop using approximated $H(x)$
- **Possible applications:**
  - ML based observation operator ("surrogate observation operator")
  - The errors from the ML observation operator can be difficult to estimate
  - DOEE can be used to estimate the errors from the ML observation operator
  - DOEE + evolving-Gaussian method can be applied in 4D-Var.

Conclusions

- We propose a new way, the evolving-Gaussian method, to incorporate general non-parametric observation error pdfs into the variational methods.

- The evolving-Gaussian method does not require revising the cost-function, and therefore it is easy to implement in a full-scale weather prediction system without adding much computational cost.

- We have implemented the evolving-Gaussian method for the all-sky microwave radiances in IFS-ECMWF, resulting in improved short-term forecasts of the lower-tropospheric cloud, water vapor and precipitations, especially in the tropics.

- The evolving-Gaussian method, together with the non-Gaussian observation error estimation technique DOEE, can potentially address the issues regarding the complicated errors in the observation operators.

Contact Info

- Feel free to send me an email for any questions, comments or more info about DOEE/the evolving-Gaussian method!

- Emails

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  - ch0683@princeton.edu